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THE TEMPERATURE DISTRIBUTION IN AN INFINITE MEDIUM RESULTING FROM A LINE SOURCE OF FINITE DURATION*

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NOMENCLATURE

- c, heat capacity;
- C_n, constant;
- $g(r)$, temperature distribution function at time t_0;
- k, thermal conductivity;
- M_n, solution function;
- p, period of heat generation;
- Q', amount of energy generated instantaneously per unit length;
- Q, amount of heat generated per unit length per unit time;
- r, radial position;
- t, time;
- T, temperature;
- ρ, density;
- η, transformation variable.

INTRODUCTION

THE PROBLEM treated in this work is that of finding the temperature distribution in an infinite medium during and following the generation of energy over a finite time period by an infinite line source. If the solution to a unit instantaneous line source is known and integrable, the temperature distribution resulting from a line source can readily be found by integration. The solution to a unit instantaneous line source is given in Carslaw and Jaeger [1] as obtained by Green's functions. It is a simple expression but is not always easily integrable, especially if the heat generation rate varies with time

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SOLUTION TO LINE SOURCES AND SINKS IN AN INFINITE MEDIUM

The energy equation for the region around an instantaneous line source generated at time $t = t'$ can be written as

$$\rho c \frac{\partial T}{\partial t} = \frac{k}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right), \quad t > t' \tag{1}$$

with boundary conditions

$$T(t, \infty) = 0 \tag{2a}$$

$$\frac{\partial T}{\partial r}(t, o) = 0 \tag{2b}$$

and global conservation of energy expressed as

$$2\pi \int_0^\infty \rho c T r \mathrm{d}r = Q' \tag{2c}$$

where Q' is defined as the strength of the source.

The solution to the above equations can be given in the form of a series (2)

$$T(t, r) = \sum_{n=0}^{\infty} \frac{C_n M_n \exp(-\eta^2/2)}{t^{n+1}\eta} \tag{3}$$

where the C_n are constant coefficients and

$$\eta = r\left(\frac{\rho c}{2kt}\right)^{\frac{1}{2}} \tag{3a}$$

$$M_n = \sum_{j=0}^{n} \frac{(-1)^j n! \eta^{2j+1}}{2^j (n-j)! j!^2}. \tag{3b}$$

The constants, C_n, in equation (3) can be evaluated if it is given that at a certain time, $t = t_0$, the temperature distribution function is $g(r)$, where $g(r)$ is an even polynomial expression of $\exp(-br^2/2)$.

To evaluate the constants, we make use of the following properties of M_n

$$\int_0^\infty \frac{M_n M_m}{\eta} \exp(-\eta^2/2)\,d\eta = 0 \qquad n = m \tag{4}$$

$$= 1 \qquad n = m. \tag{5}$$

The proof of the above is given in [2]. Thus, we may write

$$g(r) = \sum_{n=0}^{\infty} \frac{C_n M_n \exp(-\eta^2/2)}{t_0^{n+1}\,\eta} \tag{6}$$

where η is evaluated at $t = t_0$.

Multiplying the above by M_n and integrating between the limits 0 and ∞, we get the expression

$$\frac{C_n}{t_0^{n+1}} = \int_0^\infty M_n g(r)\,d\eta. \tag{7}$$

For instance, if $g(r) = \exp(-\rho c r^2/4kt'_0)$, the solution for C_n is

$$C_n = t_0^{n+1} \int_0^\infty M_n \exp(-\rho c r^2/4kt'_0)\,d\eta$$

$$= t_0^{n+1} \int_0^\infty M_n \exp(-t_0\eta^2/t_0'^2)\,d\eta$$

$$C_n = t_0^{n+1} \sum_{j=0}^{n} \frac{(-1)^j n!}{(n-j)!\,j!\,(t_0'^2/t'_0)^{j+1}}$$

and upon simplification of the above, we get

$$C_n = t'_0(t_0 - t'_0)^n \tag{8}$$

the expression for T becomes

$$T = t'_0 \sum_{n=0}^{\infty} \frac{(t_0 - t'_0)^n}{t^{n+1}} \times \sum_{j=0}^{n} \frac{(-1)^j n!}{(n-j)! j!^2} \left(\frac{\eta^2}{2}\right)^j \exp(-\eta^2/2). \tag{9}$$

It is to be noted that when $t_0 = t'_0$, the solution reduces to [since $g(r)$ is dimensionless so is T]

$$T = t'_0 \frac{\exp(-\eta^2/2)}{t}.$$

This is the solution of a simple, instantaneous line source of strength $4\pi k t'_0$ which was generated at time $t = t'_0$. This means that equation (9) is the solution of a source which is generated at time $t' = t_0 - t'_0$. Taking t' as the time elapsed before the instantaneous source is generated, we get the following expression for a simple unit source.

$$T(t,r) = \frac{1}{4\pi k} \sum_{n=0}^{\infty} \frac{t'^n}{t^{n+1}} \exp(-\eta^2/2) \frac{M_n}{\eta}. \tag{10}$$

The above solution is equivalent to the solution of [1] which can be written as

$$T(t,r) = \frac{1}{4\pi k} \frac{\exp[-\rho c r^2/4k(t-t')]}{t - t'} \tag{11}$$

The proof of equivalence is given in [2].

The advantage of equation (10) over equation (11) lies in the integrability of the former with respect to t'. The importance of this property can be seen in the following section.

APPLICATION OF SERIES SOLUTION OF AN INSTANTANEOUS HEAT SOURCE

If the heat source is of finite duration, p, and strength $Q(t')$ per unit length, the solution to the temperature distribution is the summation of the effects of the heat source at differential increments Δt, each of which may be considered as an instantaneous line source generated at time t'. The expression for T is

$$T(t,r) = \frac{1}{4\pi k} \int_0^t Q(t') \times \sum_{n=0}^{\infty} \frac{t'^n}{t^{n+1}} \exp(-\eta^2/2) \frac{M_n}{\eta}\,dt'. \tag{12}$$

The above is easily integrated when Q is a polynomial expression of t'^i where i is arbitrary. Thus, when $Q = Bt'^i$, we get upon integration of the above

$$T(t,r) = \frac{B}{4\pi k} t^i \sum_{n=0}^{\infty} \frac{1}{n+1+i} \frac{M_n}{\eta} \exp/-\eta^2/2), \quad t \leqslant p \qquad i > -1 \tag{13a}$$

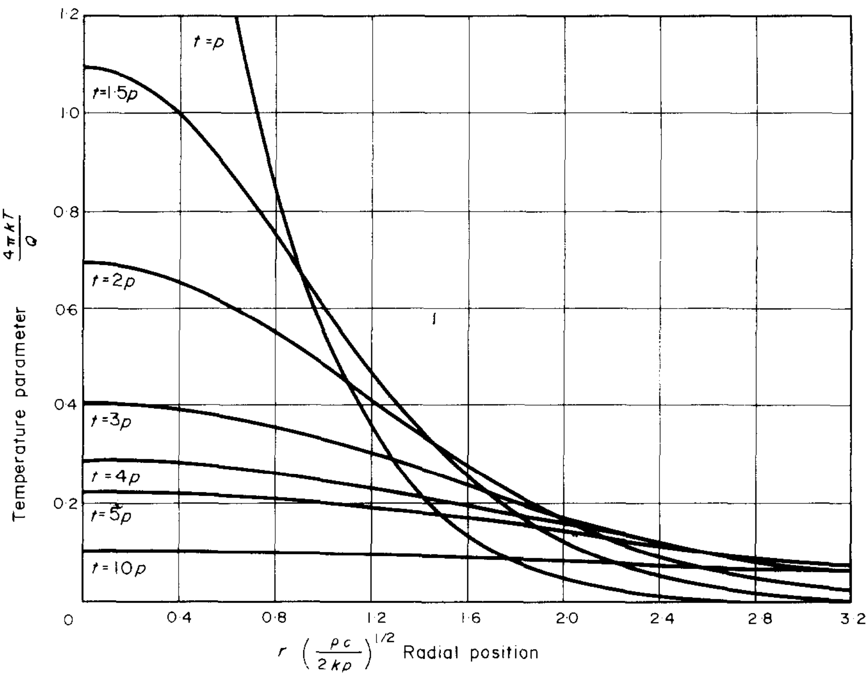

FIG. 1. Temperature distribution at various times after the end of heat generation.

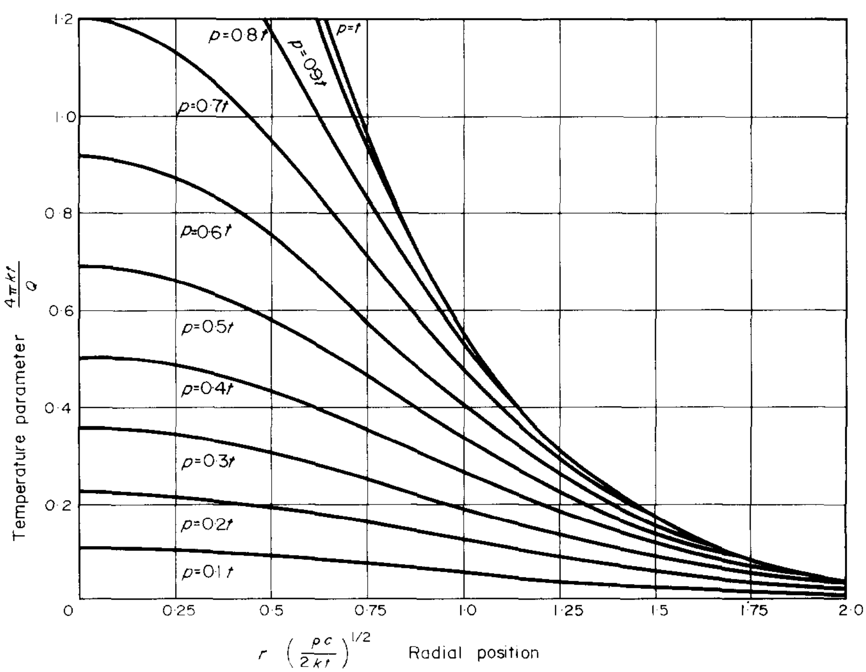

FIG. 2. Temperature distribution at a given instant in time following the end of heat generation over various periods.

$$T(t,r) = \frac{B}{4\pi k} p^i \sum_{n=0}^{\infty} \frac{1}{n+1+i} \times \left(\frac{p}{t}\right)^{n+1} \frac{M_n}{\eta} \exp(-\eta^2/2), \quad t > p \quad i > -1 \tag{13b}$$

where

$$M_n = \sum_{j=0}^{n} \frac{(-1)^j n!}{(n-j)! j!^2} \frac{\eta^{2j+1}}{2^j}.$$

It can be shown that equation (13a) converges for $t, r > 0$. The convergence of (13b) can easily be shown by comparison with the geometric series.

The temperature distribution may also be evaluated using the integral expression of equation (11). Thus

$$T = \frac{B}{4\pi k} \int_0^t \frac{t'^i \exp[-\rho c r^2/4k(t-t')]}{t-t'} \mathrm{d}t', \quad t \leqslant p \quad i > -1 \tag{14a}$$

$$T = \frac{B}{4\pi k} \int_0^p \frac{t'^i \exp[-\rho c r^2/4k(t-t')]}{t-t'} \mathrm{d}t', \quad t > p \quad i > -1. \tag{14b}$$

For $p/t \geqslant 1$, the given solutions, equations (13a) and (14a) are equivalent to the classical exponential integral solution for a continuous line source. For $p/t < 1$, the solution is that of a continuous source which then decays when the source is shut off.

For the case where $i = 0$ (constant heat generation), the long and short time expansions of equation (14a) are given on p. 262 of [1]. Equations (14a) and (14b) can also be integrated by a term by term integration of the series expansion of the integrand. The resulting expression for equation (14a), for $t \leqslant p$, is found to be easier to use than equation (13a) because the former converges faster. For the same reason, equation (13b) for $t > p$, is more convenient to use than equation (14b). Furthermore, the involved numerical integration for various values of p as given by equation (14b) is avoided by using the more accessible parametric representation of equation (13b). The results of the integration of the latter at various values of p/t for the important case $i = 0$, is shown in Figs. 1 and 2.

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POOL BOILING IN DILUTE NON-AQUEOUS POLYMER SOLUTIONS

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NOMENCLATURE

A, area of flat plate boiling surface, [ft^2];
$\bar{M}_v$, M, average polymer molecular weight, measured by a viscometric method; and molecular weight in general;
ppm, solute concentration, in parts per million by weight;
Q, rate of boiling heat transfer [Btu/h];
ΔT, T_{plate}—177°, driving force for boiling cyclohexane at 1 atm [°F].